Elliptic Curve Cryptography on Embedded Devices

Scalar Multiplication and Side-Channel Attacks

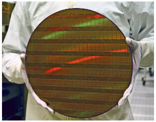
Vincent Verneuil[1,2]

[1] Inside Secure
[2] Institut de Mathématiques de Bordeaux

Séminaire Arithmétique et Théorie de l'Information
Institut de Mathématiques de Luminy
01 / 2011

Outline

Inside Secure in (very) short



Manufacturer

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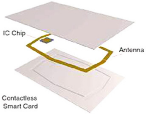

Chip Embedder

→

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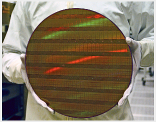

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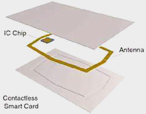

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inside SECURE

Outline

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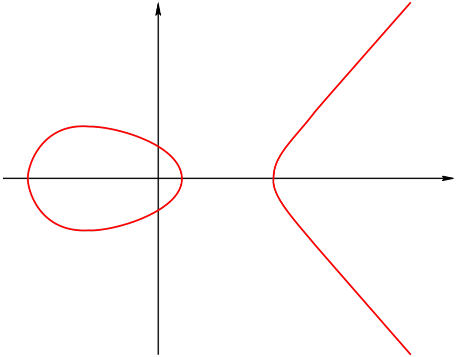
1. Elliptic Curve Cryptography
   - Generalities
   - Protocols
   - Points Representation and Formulas
   - Scalar Multiplication Algorithms
2. Side-Channel Analysis
   - Introduction
   - Simple Side-Channel Analysis
   - Differential Side-Channel Analysis
   - Fault Analysis
3. Countermeasures
   - SSCA Countermeasures
   - DSCA Countermeasures
   - FA Countermeasures
4. Conclusion

Elliptic Curve Equation

Considering a field $\mathbb{F}_p$, $p > 3$,
the points (x, y) of $\mathcal{E}/\mathbb{F}_p : y^2 = x^3 + ax + b$
and the "point at infinity" $\mathcal{O}$ form a group.



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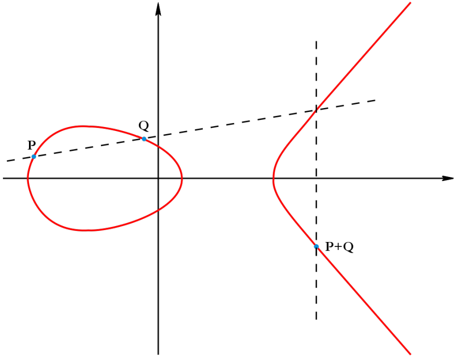

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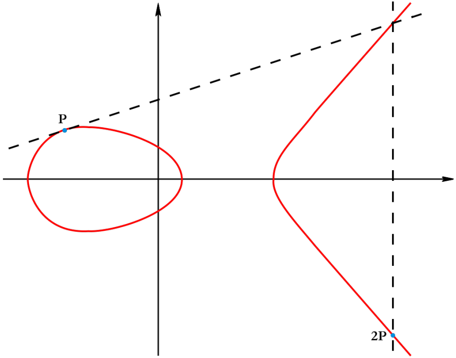

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Given P in $\mathcal{E}(\mathbb{F}_p)$ and $\alpha \cdot P$, $1 \leq \alpha \leq \#\mathcal{E}(\mathbb{F}_p)$, find α ?

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Security	2^{80}	2^{112}	2^{128}	2^{192}
ElGamal p/q	160/1024	224/2048	256/3072	384/8192
RSA	1024	2048	3072	8192
ECC	160	224	256	384

Keylengths for roughly equivalent security

Two Levels Arithmetic

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- additive law
- point additions and doublings

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Base field

- $\mathbb{F}_p$: equivalence classes of integers modulo p
- additive and multiplicative laws
- modular additions and multiplications

Embedded Devices Constraints

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- At the points group level (scalar multiplication algorithm)

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Expensive operations

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- Addition (A) A/M $\approx$ 0.2 on most smart cards
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Outline

Elliptic Curve Digital Signature Algorithm (ECDSA)

Public : $\mathcal{E}(a, b, p, n = \#\mathcal{E})$, $P \in \mathcal{E}(\mathbb{F}_p)$, H

INPUT : d and m
OUTPUT : (r, s)

Choose at random k in $[1, n-1]$
$P_1 \leftarrow k \cdot P$
$r \leftarrow x_{P_1} \mod n$
If $r \equiv 0 \mod n$ restart from the beginning
$s \leftarrow k^{-1}\left(H(m) + dr\right) \mod n$
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$$d = \frac{s \cdot k - H(m)}{r} \mod n$$

Elliptic Curve Diffie-Hellman (ECDH) Key Exchange

$$\mathcal{E}(a,b,p,n),\ P \in \mathcal{E}(\mathbb{F}_p)$$

Alice		Bob
Choose at random $a \in [1, n-1]$		Choose at random $b \in [1, n-1]$
$P_a = a \cdot P$	$\longrightarrow$	P_a
P_b	$\longleftarrow$	$P_b = b \cdot P$
$P_{ab} = a \cdot P_b$		$P_{ab} = b \cdot P_a$

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Other standards (ANSI, ISO, IEEE, SECG) $\rightarrow$ NIST curves

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Affine Representation

A point of the curve $\mathcal{E} : y^2 = x^3 + ax + b$ is represented as (x, y).

No representation for $\mathcal{O}$

Add. : 1I + 2M + 1S, Doubl. : 1I + 2M + 2S

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Homogeneous Projective Representation

A point is represented by an equivalence class $(X : Y : Z)$.

$(X : Y : Z)$ and $(\lambda X : \lambda Y : \lambda Z)$, $\lambda \neq 0$ represent the same point

$\mathcal{O} = (0 : 1 : 0)$

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Aff. $\rightarrow$ Hom. conversion :

$$(x, y) \rightarrow (x : y : 1)$$

Hom. $\rightarrow$ Aff. conversion :

$$(X : Y : Z \neq 0) \rightarrow (X/Z, Y/Z)$$

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Add. : 12M + 2S, Doubl. : 6M + 6S

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Add. : 11M + 5S, Doubl. : 2M + 8S

Modified Jacobian Projective Representation

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Faster doubling than Jacobian projective : 3M + 5S
But slower addition : 13M + 7S

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Double & Add Algorithm

Left-to-Right

INPUT : $P \in \mathcal{E}(\mathbb{F}_p)$,
$k = (k_{\ell-1} \dots k_1 k_0)_2$

OUTPUT : $k \cdot P$

$Q \leftarrow \mathcal{O}$

For i from $\ell - 1$ to 0 do
  $Q \leftarrow 2Q$
  If $k_i = 1$ then
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On average :

$$\ell \cdot \text{dbl} + \frac{\ell}{2} \cdot \text{add}$$

NAF Multiplication

NAF Representation

Signed binary representation.
Minimize the number of non-zero digits (1/3 vs 1/2).

Example :

$$187 = 10111011^{(2)} = 10\bar{1}000\bar{1}0\bar{1}^{(NAF)}$$

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Interest

- Minimize the number of additions
- $P \to -P$ is cheap : $(X : Y : Z) \to (X : -Y : Z)$

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Cost :

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- R in modified Jacobian coordinates

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Precompute $3P, 5P, \ldots$ to process several scalar bits at a time.

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Co-Z Addition

Euclidean Addition Chains [Meloni, WAIFI 2007]
Co-Z binary ladder [Goundar, Joye & Miyaji, CHES 2010]

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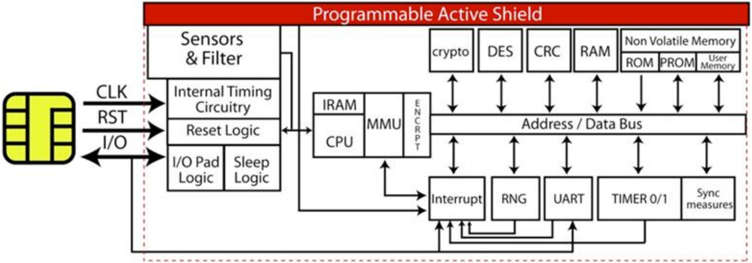
A chip in details



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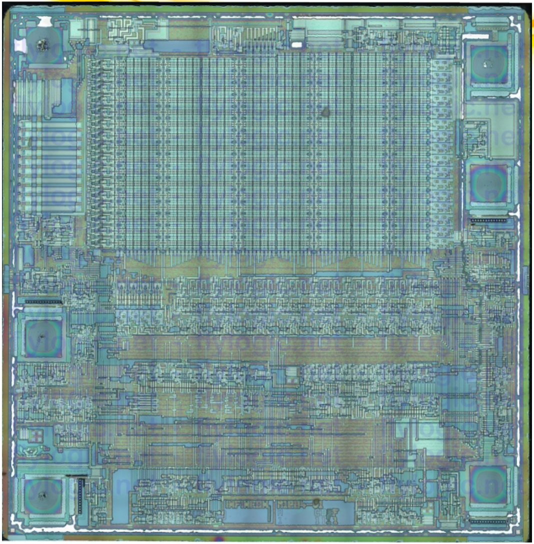
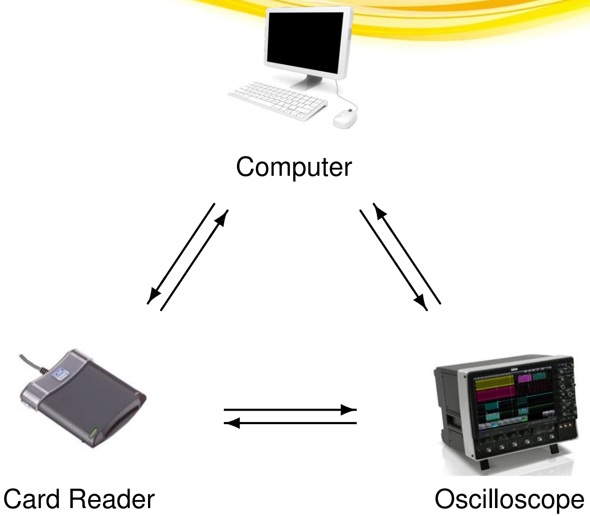


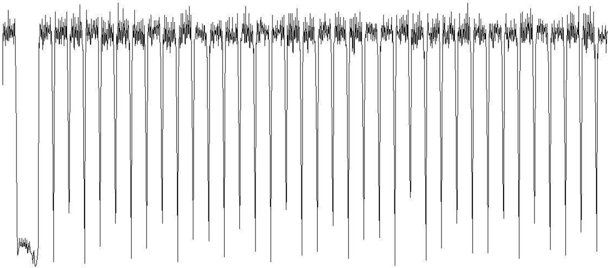
Attack Bench

Non Invasive Attacks



Simple Analyse Example

Leakage on Performed Operations



RSA exponentiation

Simple Analyse Example

Leakage on Manipulated Data

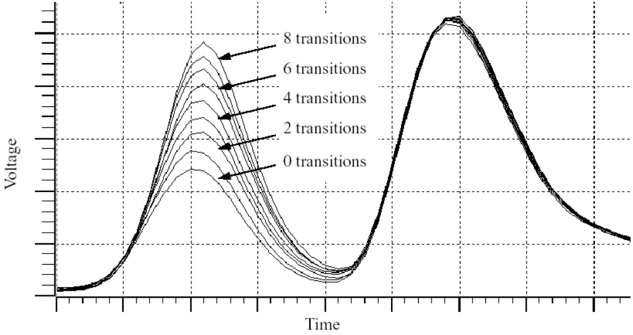


FIGURE 2. Number of Bit Transitions versus Power Consumption
These results show how the data effects the power levels. The nine overlayed waveforms correspond to the power traces of different data being accessed by an LDA instruction. These results were obtained by averaging the power signals across 500 samples in order to reduce the noise content. The difference in voltage between i transitions and i+1 transitions is about 6.5 mV.

Milestones

- Timing Attacks [Kocher, *Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems*, Crypto 1996]
- Fault Attacks [Boneh et al., *On the Importance of Checking Cryptographic Protocols for Faults*, Eurocrypt 1997]
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- DFA on ECC [Biehl et al., *Differential Fault Attacks on Elliptic Curve Cryptosystems*, Crypto 2000]
- DPA on RSA [den Boer et al., *A DPA Attack Against the Modular Reduction within a CRT Implementation of RSA*, CHES 2002]

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- CPA [Brier et al., *Correlation Power Analysis with a Leakage Model*, CHES 2004]
- CPA on PK [Amiel et al., *Power Analysis for Secret Recovering and Reverse Engineering of Public Key Algorithms*, SAC 2007]

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Measure one side-channel leakage s function of t and consider the curve $s(t)$.

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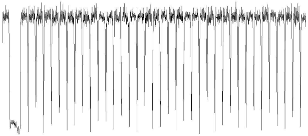

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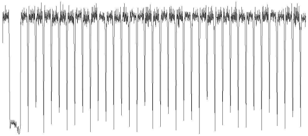

SPA/SEMA

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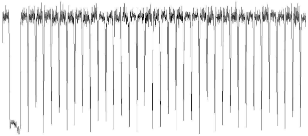
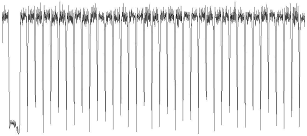


SPA/SEMA

- depicts the behavior of the chip depending on the performed operations / manipulated data

Simple Analysis Principle

Measure one side-channel leakage s function of t and consider the curve $s(t)$.



SPA/SEMA

- depicts the behavior of the chip depending on the performed operations / manipulated data
- each measure enables direct reading

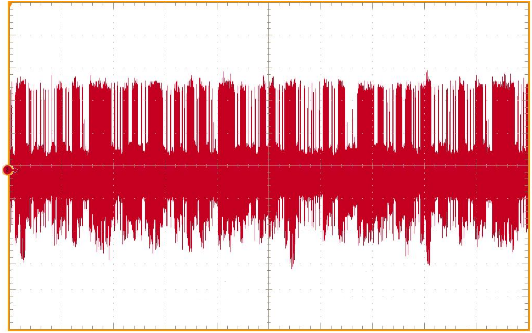
Example

Left-to-Right *Double & add* Algorithm Analysis

$Q \leftarrow \mathcal{O}$

For i from $\ell - 1$ to 0 do
  $Q \leftarrow 2Q$
  If $k_i = 1$ then
    $Q \leftarrow Q + P$

Return Q



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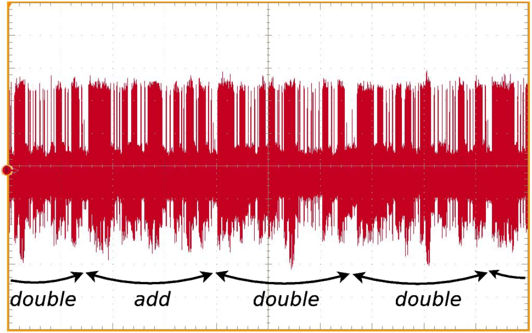

Outline

Differential Analysis Principle

Measure n times a side-channel leakage s function of t and consider the curves $s_1(t), s_2(t), \ldots, s_n(t)$.

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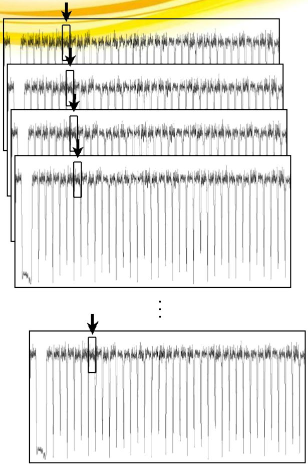

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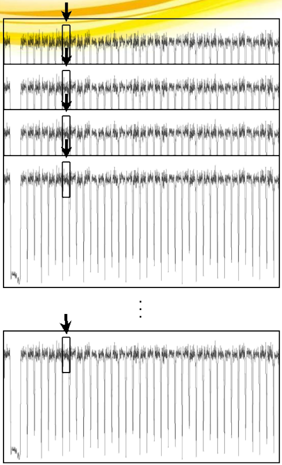

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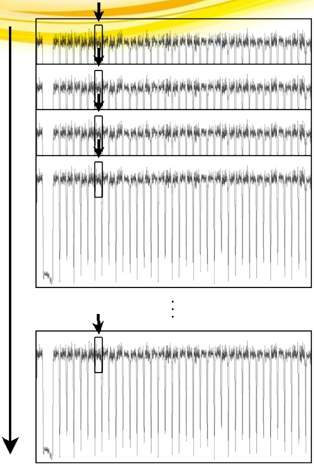
- align vertically the curves on the targeted operation
- process the curves with statistical treatment



Differential Analysis

Statistical Treatment

Depending on some known and variable input of the algorithm and of a few bits of the secret input.

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- Iterate until peaks are found

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Example

Differential Analysis

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$$C_1$$
$$C_2$$
$$\vdots$$
$$C_N$$

Differential Analysis

Statistical Treatment

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$$\begin{array}{cc} C_1 & P_1 \\ C_2 & P_2 \\ \vdots & \vdots \\ C_N & P_N \end{array}$$

Differential Analysis

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Guess : $k_i = 0$

$$\begin{array}{cc} C_1 & P_1 \\ C_2 & P_2 \\ \vdots & \vdots \\ C_N & P_N \end{array}$$

Differential Analysis

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Example

Guess : $k_i = 0$

$$\begin{array}{ccc} C_1 & P_1 & Q_1^i \\ C_2 & P_2 & Q_2^i \\ \vdots & \vdots & \vdots \\ C_N & P_N & Q_N^i \end{array}$$

Differential Analysis

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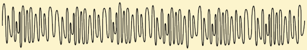

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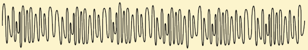
Compute $< S_0 > - < S_1 >$: 

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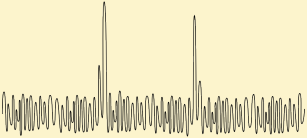

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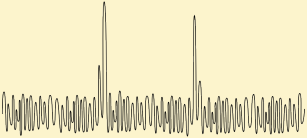

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- For each possible value (guess) :
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  - average the correlation curves and apply a threshold
- Iterate until the threshold is reached

Outline

1. Elliptic Curve Cryptography
   - Generalities
   - Protocols
   - Points Representation and Formulas
   - Scalar Multiplication Algorithms
2. Side-Channel Analysis
   - Introduction
   - Simple Side-Channel Analysis
   - Differential Side-Channel Analysis
   - **Fault Analysis**
3. Countermeasures
   - SSCA Countermeasures
   - DSCA Countermeasures
   - FA Countermeasures
4. Conclusion

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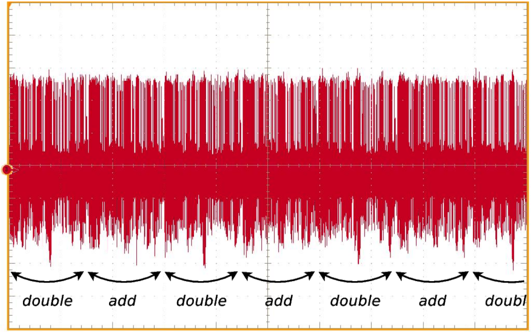

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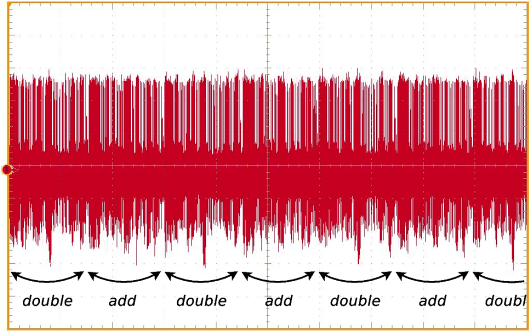

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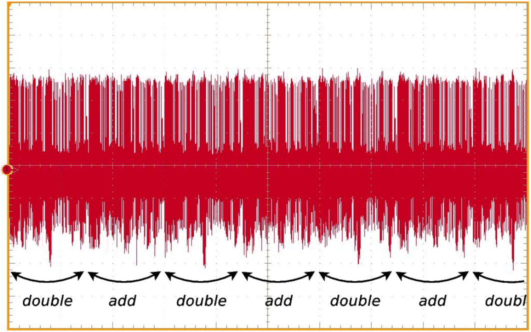

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Prone to safe errors.

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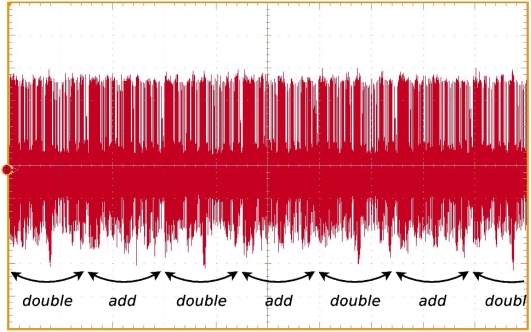

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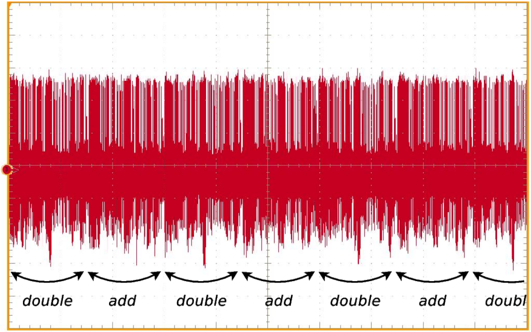

Trick :

Y_1 and Y_2 computation can be avoided.

- Brier & Joye, PKC 2002
- Izu & Takagi, PKC 2002
- Fischer et al., ePrint 2002

Unified Formulas

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Example : RSA (*square & multiply*)

- S, M, S, S, S, M, S, S, M, S, M, ...

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$\rightarrow$ Cost

Atomicity for Elliptic Curves

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Always repeat the same pattern :

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Many dummy additions/negations :(

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- Squaring
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- Addition
- Multiplication
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Atomicity Improvement

Full paper : [Giraud & Verneuil, *Atomicity Improvement for Elliptic Curve Scalar Multiplication*, CARDIS 2010]

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- Potentially applicable to every algorithm (no curve restriction)
- Prevents from the SPA at a lower cost than classical atomicity

Atomic Joye's Multiplication

Best pattern

	Add. 1	Add. 2	Dbl.
Sq.	$R_1 \leftarrow {Z_2}^2$	$R_1 \leftarrow {R_6}^2$	$R_1 \leftarrow {X_1}^2$
Add.	$\star$	$\star$	$R_2 \leftarrow Y_1 + Y_1$
Mult.	$R_2 \leftarrow Y_1 \cdot Z_2$	$R_4 \leftarrow R_5 \cdot R_1$	$Z_2 \leftarrow R_2 \cdot Z_1$
Add.	$\star$	$\star$	$R_4 \leftarrow R_1 + R_1$
Mult.	$R_5 \leftarrow Y_2 \cdot Z_1$	$R_5 \leftarrow R_1 \cdot R_6$	$R_3 \leftarrow R_2 \cdot Y_1$
Add.	$\star$	$\star$	$R_6 \leftarrow R_3 + R_3$
Mult.	$R_3 \leftarrow R_1 \cdot R_2$	$R_1 \leftarrow Z_1 \cdot R_6$	$R_2 \leftarrow R_6 \cdot R_3$
Add.	$\star$	$\star$	$R_1 \leftarrow R_4 + R_1$
Add.	$\star$	$\star$	$R_1 \leftarrow R_1 + W_1$
Sq.	$R_4 \leftarrow {Z_1}^2$	$R_6 \leftarrow {R_2}^2$	$R_3 \leftarrow {R_1}^2$
Mult.	$R_2 \leftarrow R_5 \cdot R_4$	$Z_3 \leftarrow R_1 \cdot Z_2$	$R_4 \leftarrow R_6 \cdot X_1$
Add.	$\star$	$R_1 \leftarrow R_4 + R_4$	$R_5 \leftarrow W_1 + W_1$
Sub.	$R_2 \leftarrow R_2 - R_3$	$R_6 \leftarrow R_6 - R_1$	$R_3 \leftarrow R_3 - R_4$
Mult.	$R_5 \leftarrow R_1 \cdot X_1$	$R_1 \leftarrow R_5 \cdot R_3$	$W_2 \leftarrow R_2 \cdot R_5$
Sub.	$\star$	$X_3 \leftarrow R_6 - R_5$	$X_2 \leftarrow R_3 - R_4$
Sub.	$\star$	$R_4 \leftarrow R_4 - X_3$	$R_6 \leftarrow R_4 - X_2$
Mult.	$R_6 \leftarrow X_2 \cdot R_4$	$R_3 \leftarrow R_4 \cdot R_2$	$R_4 \leftarrow R_6 \cdot R_1$
Sub.	$R_6 \leftarrow R_6 - R_5$	$Y_3 \leftarrow R_3 - R_1$	$Y_2 \leftarrow R_4 - R_2$

Outline

DPA/DEMA Protection

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Classical countermeasures :

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- Scalar blinding : $k' = k + r\#\mathcal{E}(\mathbb{F}_p)$

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DPA/DEMA Protection

Classical countermeasures :

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- Point coordinates blinding : $(X : Y : Z) = (r^2X : r^3Y : rZ)$, $r \neq 0$
- Random curve isomorphism :
  $a' \leftarrow r^4a$
  $b' \leftarrow r^6b$
  $P' \leftarrow (r^2X_P, r^3Y_P, rZ_P)$
  $Q \leftarrow (x_{Q'}/r^2, y_{Q'}/r^3)$

Outline

Fault Protection

Fault Protection

Classical countermeasures :

Fault Protection

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- Redundancy, verification...

Fault Protection

Classical countermeasures :

- Redundancy, verification...
- Verify that $P, Q \in \mathcal{E}(\mathbb{F}_p)$.

Outline

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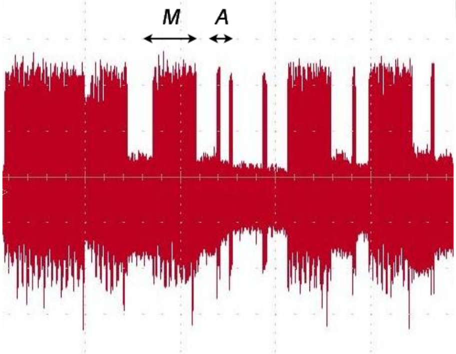
- Scalar multuplication efficiency has been extensively studied.
- Edwards curve standardization ?
- Research on side-channel attacks keeps progressing.
- Using security models for proving the resistance against attacks ?

Thank you for your attention !

Contact :

vverneuil@insidefr.com
www.math.u-bordeaux1.fr/~vverneui/

Additions Cost on a Chip



192-bit integers

A/M $\approx$ 0.2, S $=$ A, and N/M $\approx$ 0.1